Section One: Calculator-free

36% (54 Marks)

This section has **eight (8)** questions. Answer **all** questions. Write your answers in the spaces provided.

Working time: 50 minutes.

Solutions.

Question 1 **(6 marks)**

(a) A circle of radius 3 has its centre at the point $(4, -2)$.

(i) State the domain of this relation. (1 mark)

$D: \{x \in \mathbb{R}, 1 \leq x \leq 7\}$ ✓

(ii) Determine the equation of the circle in the form $x^2 + y^2 = ax + by + c$. (3 marks)

$(x-4)^2 + (y+2)^2 = 9$

$x^2 - 8x + 16 + y^2 + 4y + 4 = 9$

$x^2 + y^2 = 8x - 4y - 11$

✓ writes equation of circle
✓ correctly expands
✓ writes in correct form

(b) The graph of $x = y^2$ passes through the point $(1, q)$. Determine the value(s) of q and hence explain why y is a relation but not a function of x. (2 marks)

$1 = q^2$

$\therefore q = \pm 1$

✓ both possible values of q

Not a function as it is a one-to-many relation.

✓ explanation

Question 2 **(7 marks)**

Solve the following equations for x.

(a) $(4x-7)(x+5)=0.$ (1 mark)

$x = \frac{7}{4}$, $x = -5$ ✓ both solutions

(b) $\frac{x}{4} = \frac{3x-2}{3}.$ (2 marks)

$3x = 12x - 8$ ✓ cross multiplies

$9x = 8$ ✓ correct

$x = \frac{8}{9}$

(c) $6x = 3x^2.$ (2 marks)

$3x^2 - 6x = 0$ ✓ one correct solution

$3x(x-2) = 0$ ✓ both correct

$x = 0, \ x = 2$

(d) $x^2 + 4x - 11 = 0$ (2 marks)

$x = \frac{-4 \pm \sqrt{16 - 4(1)(-11)}}{2}$ ✓ correct method

$x = \frac{-4 \pm \sqrt{60}}{2}$ ✓ both solutions.

$x = -2 \pm \sqrt{15}$

or $x = -2 + \sqrt{15}$

$x = -2 - \sqrt{15}$

Question 3 **(8 marks)**

(a) Determine the coordinates of the

(i) y-intercept of the graph of $y = -2(x+4)^2 + 12$. (2 marks)

$y = -2(0+4)^2 + 12$ ✓ sub. in $x = 0$.

$y = -20$ $\therefore (0, -20)$ ✓ correct coordinates.

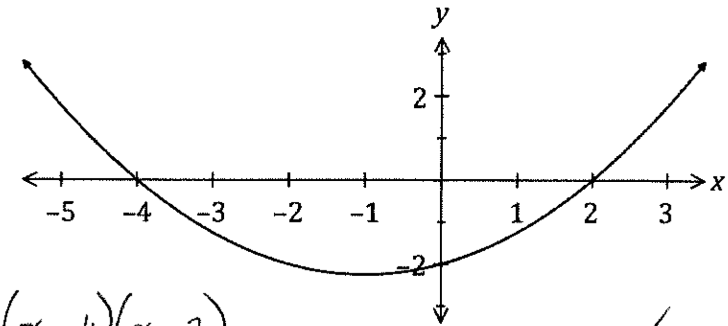
(ii) turning point of the graph of $y = (x-3)(x+1)$. (2 marks)

$x = \frac{3 + (-1)}{2} = 1$ ✓ correct x pt

$\therefore y = (1-3)(1+1)$ ✓ correct y pt.

$y = -4$ turning pt. $(1, -4)$

(b) The graph of $y = ax^2 + bx + c$ is shown below. Determine the value of the coefficients a, b and c. (4 marks)



$y = a(x+4)(x-2)$ ✓ uses root form correctly

sub. in $(0,-2)$ $\therefore -2 = a(0+4)(0-2)$ ✓ uses y int. to determine a

$a = \frac{1}{4}$

$y = \frac{1}{4}(x+4)(x-2)$ ✓ expands quadratic

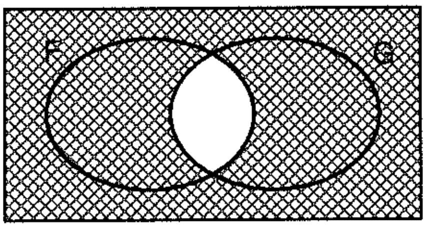
$\therefore y = \frac{1}{4}x^2 + \frac{1}{2}x - 2$ $\therefore a = \frac{1}{4}$ ✓ states $a, b + c$

$b = \frac{1}{2}$

$c = -2$

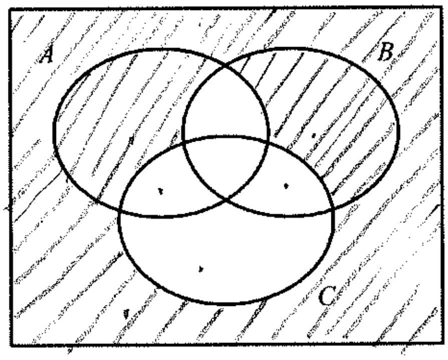
Question 4 **(7 marks)**

(a) Use set notation to describe the shaded region shown below. (1 mark)



$\overline{F \cap G}$ ✓ correct

(b) Hence or otherwise, shade $(\overline{A} \cup \overline{B}) \cap \overline{C}$ (1 mark)

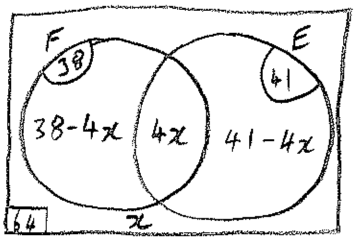


✓ correct

(c) 64 people applied for a job, of whom 38 were female and 41 had experience in a similar job. The number of females with experience was four times the number of males with no experience.

Determine

(i) the number of females with no experience. (3 marks)



$41 + x + 38 - 4x = 64$

$-3x = -15$

$x = 5$

✓ process for solving for unknown.

✓ finding unknown

$\therefore 38 - 4(5) = 18$ ✓ correct females

$\therefore$ 18 Females have no experience

(ii) the probability that a randomly chosen applicant had no experience, given that they are male. (2 marks)

$\frac{5}{26}$

✓ correct numerator

✓ correct denominator

Question 5 **(8 marks)**

(a) Briefly describe the behaviour of y for each of the following graphs, given the behaviour of x:

(i) $y = x^5$, as $x \to -\infty$. (1 mark)

$y \to -\infty$ ✓ correct

(ii) $y = \frac{1}{x}$, as $x \to \infty$. (1 mark)

$y \to 0$ ✓ correct

(iii) $y = (1-2x)^2$, as $x \to \infty$. (1 mark)

$y \to \infty$ ✓ correct

(b) The graph of $y = f(x)$ is shown below.

(i) Determine the equation of *f(x)* in the form $y = 2\sqrt{x+c} - d$ (2 marks)

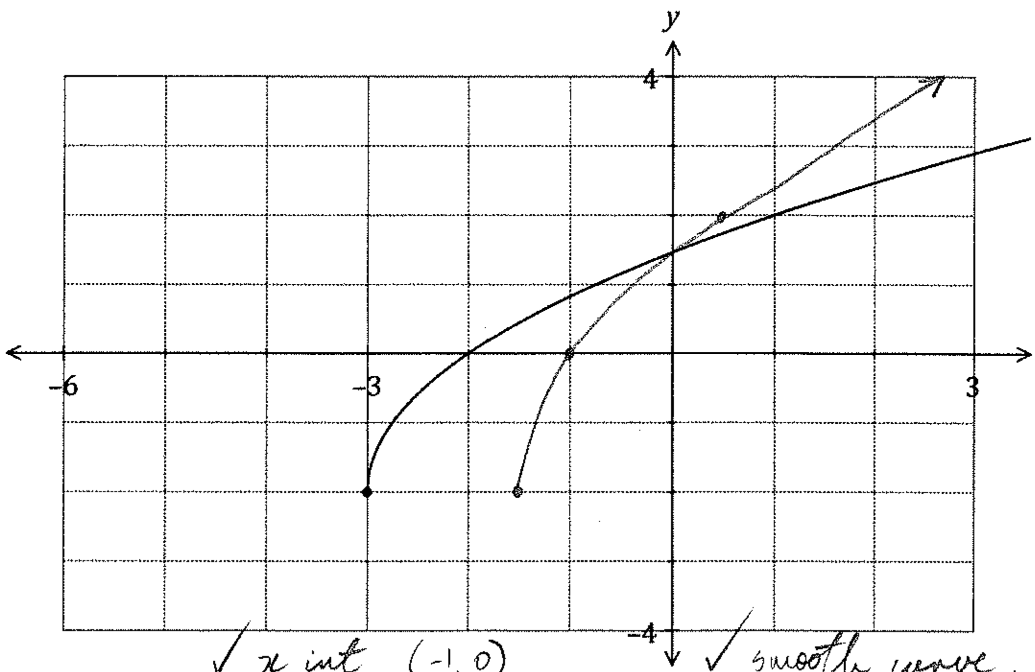
$y = 2\sqrt{x+3} - 2$

✓ c value
✓ d value

(ii) State the range of *f(x)*. (1 mark)

$\{y \in \mathbb{R};\ y \geq -2\}$ ✓ correct

(ii) Graph $y = f(2x)$ on the same set of axes. (2 marks)



✓ x int. (−1, 0) and same y int.

✓ smooth curve, starting at (−1.5, −2)

Question 6 **(6 marks)**

Let $f(x) = x^3 + 2x^2 - 11x - 12$.

(a) Identify the leading coefficient of *f(x)*. (1 mark)

1 ✓ correct

(b) Determine $f(-1)$. (1 mark)

$f(-1) = (-1)^3 + 2(-1)^2 - 11(-1) - 12$

$f(-1) = 0$ ✓ correct

(c) Solve $f(x) = 0$. (4 marks)

$$
\begin{array}{r}
x^2 + x - 12 \\
x+1 \,\big)\, \overline{x^3 + 2x^2 - 11x - 12} \\
-(x^3 + x^2) \\
\hline
x^2 - 11x - 12 \\
-(x^2 + x) \\
\hline
-12x - 12 \\
-(-12x - 12) \\
\hline
0
\end{array}
$$

✓ process for finding quadratic factor

$\therefore (x+1)(x^2 + x - 12)$ ✓ determines quadratic factor

$(x+1)(x+4)(x-3)$ ✓ factorises quadratic

$\therefore x = -1, -4$ and 3 ✓ all correct solutions

Question 7 **(7 marks)**

(a) Expand $(x + 5)^3$ (2 marks)

$x^3 + 3 \times x^2 \times 5 + 3 \times x \times 5^2 + 5^3$

$\therefore \; x^3 + 15x^2 + 75x + 125$

✓ use of binomial coefficients
✓ correct expansion

(b) Complete the row of Pascal's triangle that starts 1, 5, 10, ... and express the sum of the numbers in this row as a power of 2. (1 mark)

$1 \quad 5 \quad 10 \quad 10 \quad 5 \quad 1 = 2^5$ ✓ correct

(c) Hence, determine the coefficient of

(i) the x^4 term in the expansion of $(x + 1)^5$. (1 mark)

$5 \times x^4 \times 1^1$

$= 5x^4$

$\therefore \; 5$ ✓ correct

(ii) the x^3 term in the expansion of $(2 - 3x)^5$. (3 marks)

${}^5C_3 \times (2)^2(-3x)^3$

$10 \times 4 \times (-27)x^3$

$= -1080x^3$

$\therefore \; -1080$

✓ correct three factors of term
✓ expands each factor
✓ states coefficient

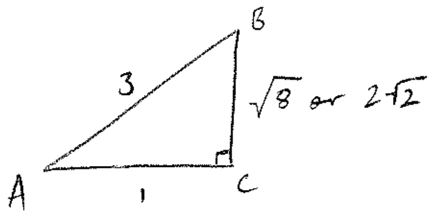
Question 8 **(5 marks)**

(a) Evaluate $\sin\left(\frac{35\pi}{42}\right)$. (2 marks)

$\sin\frac{5\pi}{6} = \sin\frac{\pi}{6}$ ✓ simplifies correctly

$\therefore = \frac{1}{2}$ ✓ correct exact value

(b) An acute angle A exists such that $\cos A = \frac{1}{3}$. Show that $\sin A = \frac{2\sqrt{2}}{3}$ and hence, determine the value of $\tan A$. (3 marks)



$BC = \sqrt{3^2 - 1^2}$

$= \sqrt{8}$

$= 2\sqrt{2}$ ✓ use of pyth. to find opp. side to A

$\therefore \sin A = \frac{opp}{hyp} = \frac{2\sqrt{2}}{3}$ ✓ use of sin ratio

$\therefore \tan A = \frac{2\sqrt{2}}{1}$ ✓ finds tan A

$\therefore \tan A = 2\sqrt{2}$

End of questions

Supplementary page

Question number: _________

Supplementary page

Question number: ________

Section Two: Calculator-assumed **64% (96 Marks)**

This section has **thirteen (13)** questions. Answer **all** questions. Write your answers in the spaces provided.

Working time: 100 minutes.

Question 9 **(6 marks)**

(a) The points A and B have coordinates $(4, -6)$ and $(5, 8)$ respectively. If B is the midpoint of A and C, determine the coordinates of C. (2 marks)

$\frac{4+x}{2} = 5$ $\frac{-6+y}{2} = 8$ ✓ uses midpt. equation

$\therefore x = 6$ $\therefore y = 22$ ✓ x coord.

✓ y coord.

$C(6, 22)$

(b) x and y are linearly related variables such that the points $D\ (5p, -q)$ and $E\ (2q, 3p)$ lie on $y = mx + c$. Determine the relationship for q in terms of p, if:

(i) the gradient of the line is 2. (2 marks)

$2 = \frac{3p+q}{2q-5p}$ ✓ uses gradient formula

$4q - 10p = 3p + q$

$3q = 13p$ $\therefore q = \frac{13p}{3}$ ✓ writes correct q in terms of p

(ii) Hence, determine the value of the y intercept if $p = 3$. (2 marks)

$\therefore q = \frac{13(3)}{3} = 13$ $\therefore E(26, 9)$

✓ Determines q and a coord.

$y = 2x + c$

$9 = 2(26) + c$

$\therefore c = -43$ $\therefore$ y int is $(0, -43)$ ✓ states y int.

Question 10 **(8 marks)**

A random sample of 121 passengers arriving at an airport were asked to complete a brief survey. They were asked to categorise their main place of residence as Australia or overseas and the main purpose of their travel as work, holiday or other. It was found that

- half of the 84 passengers who resided overseas were on holiday
- 14 passengers were on holiday and resided in Australia
- of the 27 who were travelling for other reasons, 11 more resided overseas than in Australia.

(a) Use the above information to complete the two-way table below. (3 marks)

	Work	Holiday	Other	Total
Australia	15	14	8	37
Overseas	23	42	19	84
Total	38	56	27	121

✓ holiday column
✓ other column
✓ table correct

(b) If one passenger was selected at random from those surveyed, determine the probability (to 4 decimal places)

(i) that the main purpose of their travel was work. (1 mark)

$\frac{38}{121} \approx 0.3140$ ✓ correct

(ii) that they resided overseas, given that the main purpose of their travel was work. (1 mark)

$\frac{23}{38} \approx 0.6053$ ✓ correct

(iii) that the main purpose of their travel was work, given that they resided in Australia. (1 mark)

$\frac{15}{37} \approx 0.4054$ ✓ correct

(c) Explain whether the survey indicates that purpose of travel appears to be independent of main place of residence for these passengers. (2 marks)

Purpose of travel is <u>not</u> independent of residence

$P(T_W | R_A) \neq P(T_W)$ ✓ reason

$0.4054 \neq 0.3140$ ✓ probabilities

Question 11 **(7 marks)**

A positive integer less than 11 is chosen at random.

The outcome sets for events O, T and S are such that:
$O = \{odd\ numbers\} = \{1, 3, 5, 7, 9\}$
$T = \{triangular\ numbers\} = \{1, 3, 6, 10\}$ and
$S = \{square\ numbers\} = \{1, 4, 9\}$.

(a) List the elements of the following sets:

(i) $O \cap T$. (1 mark)

$\{1, 3\}$ ✓ correct

(ii) $T \cup (O \cap S')$. (2 marks)

$\{1, 3, 5, 6, 7, 10\}$

✓ 4 correct
✓ all correct

(b) Determine

(i) $\mathrm{n}(O \cap S \cap T')$. (1 mark)

1 ✓ correct

(ii) $P(O' \cap (T \cap S))$. (1 mark)

$\frac{0}{11} \therefore 0$ ✓ correct

(iii) $P(T'|(O \cup S))$. (2 marks)

$\frac{4}{6}$

✓ numerator
✓ denominator

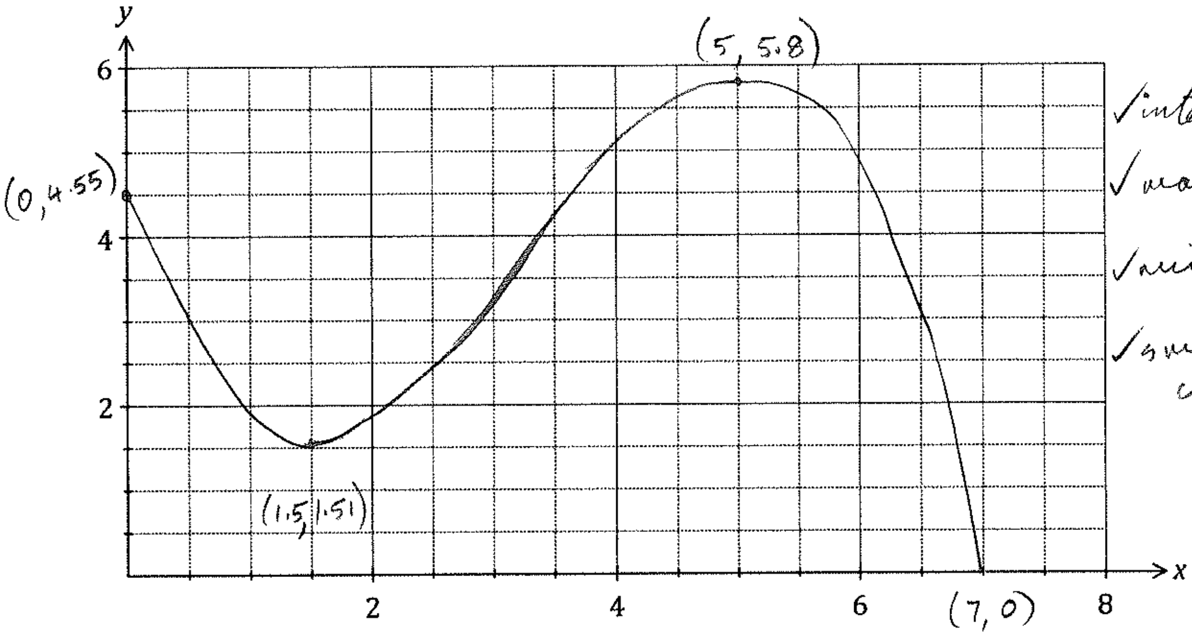
Question 12 **(8 marks)**

The distortion of a signal, D, can be modelled by $D(x) = 4.55 - 4.5x + 1.95x^2 - 0.2x^3$, where x is the distance from the signal source in metres and $0 \leq x \leq 7$.

(a) Determine D when $x = 1$. (1 mark)

$D(1) = 1.8$ ✓ correct

(b) Draw the graph of $y = D(x)$ on the axes below. (4 marks)



✓ intercepts
✓ maximum
✓ minimum
✓ smooth curve

(c) The strength of the signal, *S*, is inversely proportional to the distance from the signal source, *x*, such that at 1.5 metres from the source, the strength is 1.5. Determine the distances at which the distortion, *D*, is equal to the signal strength, *S*. (3 marks)

$S \propto \frac{k}{x}$

$\therefore k = 1.5 \times 1.5$

$k = 2.25$

$\therefore S = \frac{2.25}{x}$

$\therefore \frac{2.25}{x} = 4.55 - 4.5x + 1.95x^2 - 0.2x^3$

$x = 1.487$ and

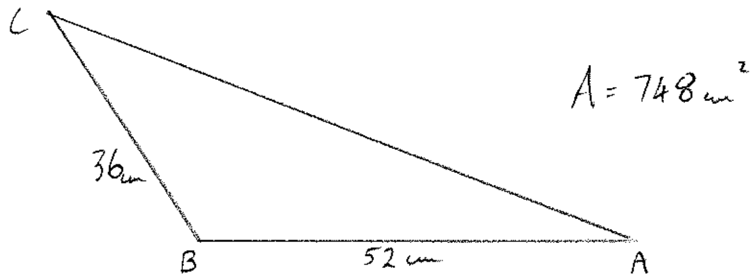
$x = 6.950$

✓ inverse proportion formula.
✓ solving signal and distortion eq.s
✓ both solutions.

Question 13 **(7 marks)**

A triangle ABC has $a = 36$ cm, $c = 52$ cm and an area of 748 cm^2.

(a) Sketch a triangle to show this information. (1 mark)



✓ correct sketch

If $\angle B$ is an obtuse angle in the triangle

(b) Determine the size of $\angle B$. (2 marks)

$748 = \frac{1}{2} \times 36 \times 52 \times \sin B$

$\therefore \angle B = 126.95°$

✓ uses area formula

✓ correct angle

(c) Show that $b \approx 79$ cm. (2 marks)

$b^2 = 52^2 + 36^2 - 2(52)(36)\cos 126.95$

$\therefore b = 79.06$ cm

$= 79$ cm

✓ uses appropriate equation

✓ solves correctly

(d) Show that $\angle C \approx 32°$. (2 marks)

$\frac{52}{\sin C} = \frac{79.06}{\sin 126.95}$

$\angle C = 31.71°$

$\therefore \; = 32°$

✓ uses appropriate equation

✓ solves correctly

Question 14 **(8 marks)**

Two events, A and B, have probabilities $P(A) = 0.4$ and $P(B) = 0.65$.

(a) Determine $P(A \cap B)$ in each of the following cases:

(i) A and B are independent. (1 mark)

$P(A) \times P(B) = 0.4 \times 0.65$

$= 0.26$ or $\frac{13}{50}$

✓ correct

(ii) $P(A \cup B) = 0.8$. (2 marks)

$P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$0.8 = 0.4 + 0.65 - P(A \cap B)$

$\therefore P(A \cap B) = 0.25 = \frac{1}{4}$

✓ uses appropriate rule

✓ correct

(iii) $P(A|(A \cup B)) = \frac{4}{9}$. (3 marks)

$\frac{4}{9} = \frac{0.4}{P(A \cup B)}$

$\therefore P(A \cup B) = 0.9$

$\therefore P(A \cap B) = 0.4 + 0.65 - 0.9$

$= 0.15$ or $\frac{3}{20}$

✓ uses appropriate rules

✓ forms equation

✓ correct

(b) Is it possible that A and B are mutually exclusive events? Explain your answer. (2 marks)

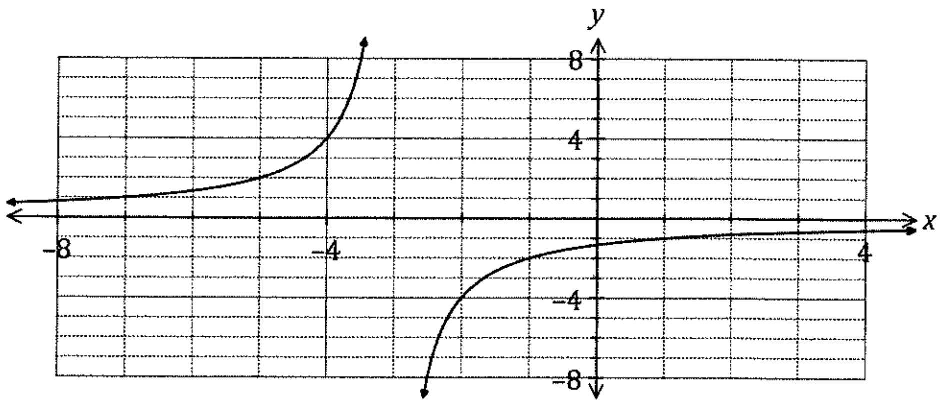
No, as $P(A) + P(B) > 1$, $\therefore$ not possible.

✓ states no (with reason)

✓ correct reason

Question 15 **(7 marks)**

The graph of $y = f(x)$ is shown below where $f(x) = \dfrac{-a}{x-b}$.



(a) The hyperbola shown above has two asymptotes. State their equations. (2 marks)

$x = -3$ ✓ one correct

$y = 0$ ✓ both correct

(b) State the values of constants a and b. (2 marks)

$b = -3$ ✓ value of b

$\therefore f(x) = \dfrac{-a}{x+3}$ ✓ value of a

sub. in $(-1, -2)$ $-2 = \dfrac{-a}{-1+3}$

$\therefore a = 4$

(c) Describe how to transform the graph of $y = f(x)$ to obtain the graph of $y = f(x) + 1$ and state the domain and range of the transformed function. (3 marks)

Translate the graph 1 unit up.

Domain: $\{x \in \mathbb{R};\ x \neq -3\}$ ✓ transformation

Range: $\{y \in \mathbb{R};\ y \neq 1\}$ ✓ domain

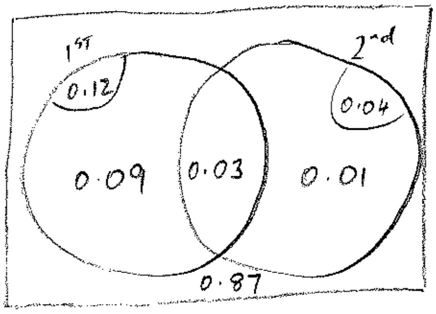
✓ range

Question 16 **(7 marks)**

An examination consisted of two papers, one of which was much harder than the other. 12% of candidates gained a distinction in the first paper (event A) and 4% gained a distinction in the second paper (event B) whilst 87% did not gain a distinction in either paper.

(a) Using an appropriate diagram, determine the probability that a randomly chosen candidate

(i) gained a distinction in both papers. (3 marks)



$P(1^{st} \cap 2^{nd}) = 0.03$

✓ given information in diagram

✓ completed diagram

✓ correct value

(ii) gained a distinction in one paper but not the other. (1 mark)

$0.09 + 0.01 = 0.1$

✓ correct value

(iii) gained a distinction in the second paper given that they gained a distinction in the first. (1 mark)

$\frac{0.03}{0.12} = 0.25$

✓ correct value

(b) State, with justification, whether events A and B are independent. (2 marks)

$P(1^{st}) \times P(2^{nd}) = P(1^{st} \cap 2^{nd})$

$0.12 \times 0.04 \neq 0.03$

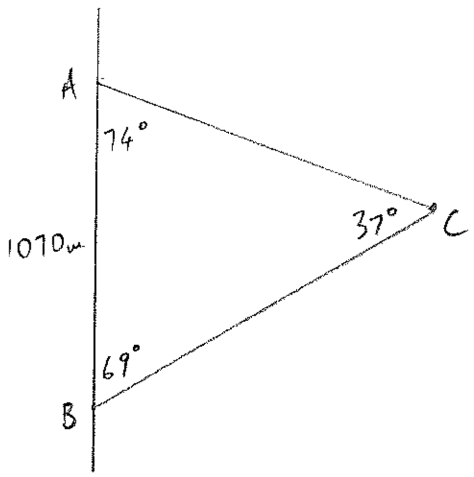
$\therefore$ not independent.

✓ not independent (with reason)

✓ correct use of rule

Question 17 **(8 marks)**

(a) A and B are two points on a coastline, and C is a point at sea. The points A and B are 1070m apart. The angles CAB and CBA have magnitudes of 74° and 69° respectively. Find the distance from C to A to the nearest metre. (3 marks)



$$\frac{AC}{\sin 69°} = \frac{1070}{\sin 37°}$$

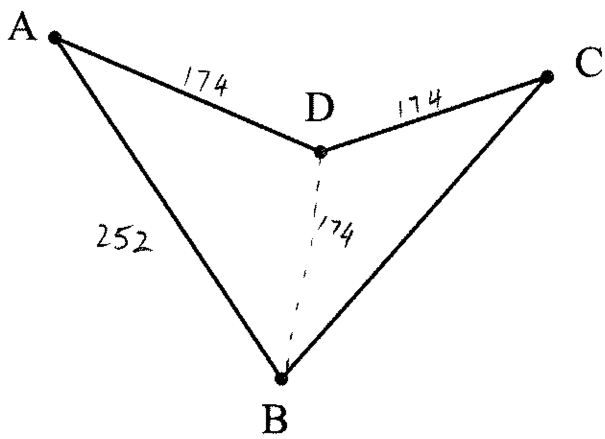
$AC = 1659.86$ m

$\therefore AC = 1660$ m

✓ uses appropriate rule
✓ correct value
✓ nearest metre

(b) Determine the area of the quadrilateral shown below given that $\angle BDC = \angle ADC$

$AB = 252$m and $AD = BD = CD = 174$m. (5 marks)



✓ uses approp. rule, finds angle ADB
✓ finds angle BDC
✓ uses area formula for Δ's
✓ correct area of one Δ
✓ correct total area.

$252^2 = 174^2 + 174^2 - 2 \times 174 \times 174 \times \cos \angle ADB$

$\therefore \angle ADB = 92.79°$

$\therefore \angle BDC = 133.60°$

$Area_{\Delta ABD} = \frac{1}{2} \times 174 \times 174 \times \sin 92.79° = 15120\text{m}^2$

$Area_{\Delta BCD} = \frac{1}{2} \times 174 \times 174 \times \sin 133.60° = 10962\text{m}^2$

26082m^2

Question 18 **(9 marks)**

(a) The equation of the axis of symmetry for the graph of $y = 3x^2 + 6x + 7$ is $x = k$. Determine the value of k, using a method that does not refer to the graph of the parabola. (2 marks)

$k = \frac{-6}{2 \times 3}$ ✓ uses $\frac{-b}{2a}$ method

$k = -1$ ✓ correct value

(b) A parabola has a turning point at $(6, -5)$ and passes through the point $(-2, -37)$.

(i) Determine the equation of the function. (2 marks)

$y = a(x-6)^2 - 5$ ✓ subs in t.p into completed square form

sub in $(-2,-37)$

$-37 = a(-2-6)^2 - 5$

$a = -\frac{1}{2}$ ✓ subs in point and finds equation

$\therefore y = -\frac{1}{2}(x-6)^2 - 5$

(ii) Show that the equation has no real zeroes (i.e. no real roots) (2 marks)

$-\frac{1}{2}(x-6)^2 - 5 = 0$ ✓ function equals zero

$(x-6)^2 = -10$

however $(x-6)^2 \neq -10$ ✓ shows 'not possible' mathematically

$\therefore$ no real roots

(c) Determine the value of the discriminant for the quadratic equation $16x^2 - 24x + 9 = 0$ and use it to explain how many solutions the equation $(x+1)(16x^2 - 24x + 9) = 0$ will have. (3 marks)

$b^2 - 4ac \Rightarrow (-24)^2 - 4(16)(9)$ ✓ uses discriminant correctly

$= 0$

$\therefore$ one solution for $16^2 - 24x + 9$

So $(x+1)(16x^2 - 24x + 9) = 0$ will therefore have <u>two</u> solutions ✓ finds one solution for quadratic

✓ explains why there are two solutions.

Question 19 **(6 marks)**

Let $p = \cos\frac{13\pi}{18}$ and $q = \sin\frac{7\pi}{36}$.

Give your answers to the following in terms of p and/or q.

(a) Write down an expression for

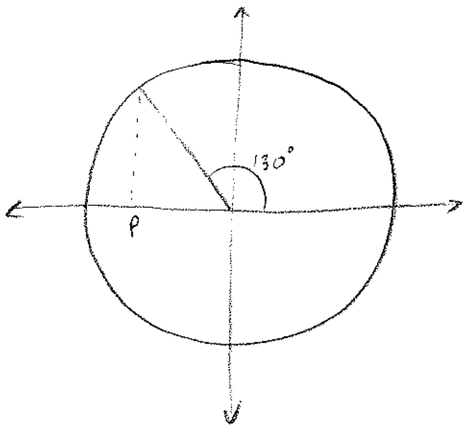
(i) $\sin\frac{29\pi}{36}$ (1 mark)

q ✓ correct

(ii) $\cos\frac{5\pi}{18}$ (1 mark)

$-p$ ✓ correct

(b) Using your understanding of the unit circle, determine all other values of θ, within two revolutions whereby $\cos\theta = p$. Give your answers in degrees. (4 marks)



$p = \cos\frac{13\pi}{18}$, $\frac{13\pi}{18} = 130°$

$\therefore\ \theta = 230°, 490°, 590°$

✓ shows use of unit circle
✓ converts to degrees
✓ gives answer within 1 revolution
✓ gives all correct answers

Question 20 **(7 marks)**

A shelf held a collection of 22 different books, of which 5 were encyclopedias, 10 were science fiction and the rest were poetry.

A random selection of 4 books is to be made from the shelf.

(a) Determine the number of ways

(i) this can be done. (1 mark)

$^{22}C_4 = 7315$

✓ correct value

(ii) a selection can be made that will not contain any encyclopedias. (2 marks)

$^{17}C_4 \times {}^5C_0 = 2380$

✓ correct use of combinations

✓ correct value

(b) Determine the probability that

(i) the selection will only contain poetry. (2 marks)

$^7C_4 \times {}^5C_0 \times {}^{10}C_0 = 35$

✓ correct number for selection

$\therefore P = \frac{35}{7315} = \frac{1}{209} \approx 0.00478$

✓ correct probability

(ii) the selection will contain exactly one poetry book given that it does not contain any encyclopedias. (2 marks)

$^7C_1 \times {}^{10}C_3 \times {}^3C_0 = 840$

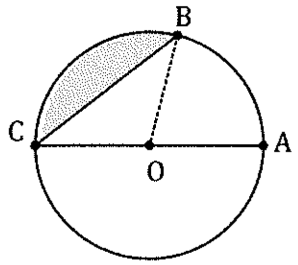
✓ correct number for selection

$\therefore P = \frac{840}{2380} = \frac{6}{17} \approx 0.3529$

✓ correct probability

Question 21 **(8 marks)**

(a) The circle shown has centre O and diameter AC of length 60 cm. Determine the shaded area given that $7 \times \angle AOB = 5 \times \angle BOC$. (4 marks)



$\angle AOB = \frac{5}{7} \angle BOC$ ✓ equation using angles

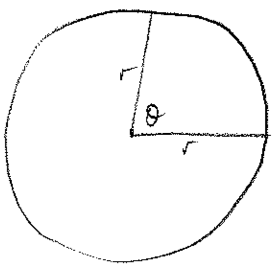
$\therefore \angle BOC + \frac{5}{7}\angle BOC = \pi$ ✓ correct angle for segment

$\therefore \angle BOC = \frac{7\pi}{12}$ or $105°$

$\therefore A = \frac{1}{2}(30)^2\left(\frac{7\pi}{12} - \sin\frac{7\pi}{12}\right)$ ✓ uses formula appropriately

$A = 390\text{ cm}^2$ ✓ correct area

(b) A sector of a circle with radius r and subtended angle θ has a perimeter of 91 cm and an area of 490 cm^2. Determine the possible values of r and θ that satisfy these conditions. (4 marks)



$91 = 2r + \theta r$ ✓ equation for perimeter

$490 = \frac{1}{2}r^2\theta$ ✓ equation for area

Solving simultaneously ✓ solves equations

$r = 28\text{ cm}$, $\theta = \frac{5}{4}$ or $68.75°$

<u>OR</u> ✓ states both sets of solutions

$r = 17.5\text{ cm}$, $\theta = \frac{16}{5}$ or $183.35°$

End of questions

Supplementary page

Question number: __________